Solved Problems: Utility and Partial Equilibrium Market Model

This module will try to solve problems on topics discussed in Module 7 and Module 8

9.1. There are ten boxes with different quantities of chocolates and ball pens.

a) Determine the rank of the bundles assuming that more is better. Give higher rank to more preferred box.

b) Represent the situation graphically

Box Name	**Quantity of**	
	Chocolate	**Ball Pen**
A	9	9
B	9	8
C	6	8
D	5	9
E	6	7
F	5	7
G	6	6
H	5	6
I	9	1
J	1	1

Solution:

a) Since more is better, therefore the box that contains more of both chocolate and ball pen will be preferred, we get the following table:

Box Name	**Quantity of**		**Total quantities of chocolate and ball pen**
	Chocolate	**Ball Pen**	
A	9	9	18
B	9	8	17
C	5	9	14
D	6	8	14
E	6	7	13
F	5	7	12
G	6	6	12
H	5	6	11
I	9	1	10
J	1	1	2

The ranks may be given as follows:

Box name	Total quantities of chocolate and ball pen	Quantity of		Rank
		Chocolate	Ball Pen	
A	18	9	9	6
B	17	9	8	5
C	14	5	9	5
D	14	6	8	5
E	13	6	7	4
F	12	5	7	3
G	12	6	6	3
H	11	5	6	2
I	10	9	1	2
J	2	1	1	1

Box J contains a total of 2 quantities (one chocolate and one ball pen). Therefore, it is the least preferred as compared to all the other boxes and is given **Rank 1**. Box I is preferred to J as it has same number of ball pens but more of chocolates. Box H is preferred to J but will be indifferent to I because, it has more of ball pens but less of chocolates. Box G has same number of ball pens as in H but more chocolate. Hence G will be preferred to H. Box F will be indifferent to G as it has more of ball pen but less of chocolate. Similarly, E is preferred to F, D is preferred to E, C is indifferent to D, B is indifferent to C and A is preferred to B with the highest rank.

The relationship between the boxes graphically represented as follows:

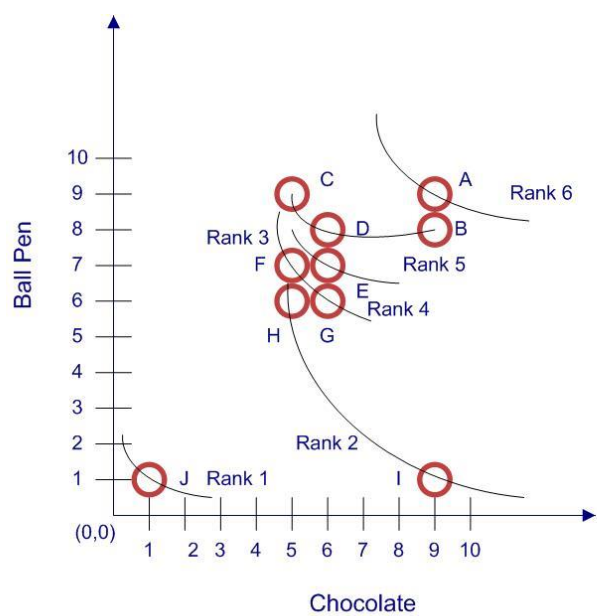

Fig: Indifference Map

9.2. The following table gives the quantities of mangoes and the total utility for a particular consumer.

Quantity of mangoes	**Total Utility (In Utils)**
1	20
2	35
3	45
4	53
5	55
6	55
7	45
8	40

a) Calculate the Marginal Utility of mangoes.
b) Does this example follow the Law of Diminishing Marginal Utility?
c) If the Marginal Utility of money (MU_m) is 4 Utils, and the price of one mango is ₹2, how many mangoes will the consumer consume?

Solution:

a) Marginal Utility of mango is the additional utility attained by consuming an additional mango. One unit of mango has a Total Utility of 20 Utils, which means that the Marginal Utility from the first mango is 20 Utils. Two mangoes give a Total Utility of 35 Utils. This means that the second mango consumed has a Marginal Utility of (35-20) = 15 Utils. The Marginal Utility for the successive units of mangoes consumed is calculated using the same concept.

This is given in the following table:

Quantity of mangoes	**Total Utility (In Utils)**	**Marginal Utility (In Utils)**
1	20	20
2	35	(35-20)= 15
3	45	(45-35)= 10
4	53	(53-45)= 8
5	55	(55-53)= 2
6	55	(55-55)= 0
7	45	(45-55)= -5
8	30	(30-45)= -15

b) By observing the Marginal Utilites, it may be concluded that as more and more units of mangoes are consumed, the utility derived from an additional mango keeps falling. The consumption of the 6^{th} Mango gives a utility of zero and consumption of 7^{th} and 8^{th} mango gives negative utility. Therefore it may be concluded that that the example follows the Law of Diminishing Marginal Utility.

c) Marginal Utility of money is defined as the utility derived by spending one unit of money, that is, ₹1= 4 utils or 4 utils = ₹1. Therefore 20 utils = 20/4= ₹5.

The Marginal Utilities in terms of money for each successive unit may be calculated. This is given in the following table:

Quantity of mangoes	**Marginal Utility (in utils)**	**Marginal Utility (in terms of money)**
(1)	**(2)**	**(3)**
1	20	20/4= 5
2	15	15/4= 3.75
3	10	10/4= 2.5
4	8	8/4= 2
5	2	2/4= 0.5
6	0	0/4= 0
7	-5	-5/4= -1.25
8	-15	-15/4= -3.75

In order to find out the number of mangoes that will be consumed, it is important to first understand that a consumer aims at maximizing satisfaction or utility. Now, the first mango gives a marginal utility in terms of money equal to ₹5 and the price paid for the mango is ₹2. Clearly, the utility derived from the first mango is very high as compared to the price paid. Therefore, the consumer will consume the first mango. The second mango gives marginal utility worth ₹3.75 that is also higher than the price paid. Therefore, the consumer will consume the second mango as well. The fourth mango has marginal utility worth ₹2 that is exactly equal to the price of the mango. In this case, the consumer may wish to consume as it gives exactly same utility as the amount spent on the mango. The fifth mango gives utility worth ₹0.5 that is less than the price paid for it. Therefore, the consumer will not want to consume the fifth mango. The sixth, seventh and eighth mangoes also give utilities that are less than the price paid for one mango.
Thus it may be concluded that the consumer will stop at the fourth mango and will consume four mangoes for the given example. In other words, according to the Cardinal Utility Theory, it may be said that the consumer will be in equilibrium when

MU (in terms of money) = Price

9.3. Given the following market model

$$D = 10 - 2P$$
$$S = -6 + 2P$$
$$D = S$$

a) Calculate the excess demand for different values of Price
b) Find market-clearing price and quantity demanded and supplied based on the concept of excess demand
c) Calculate the equilibrium price and quantity demanded and supplied by using the method of elimination and substitution.

Solution:

a) By definition, Excess Demand = Demand –Supply. The excess demand at different prices obtained by using the given equations is given in the following table:

Price (in ₹)	Demand	Supply	Excess Demand
1	8	-4	12
2	6	-2	8
3	4	0	4
4	2	2	0
5	0	4	-4
6	-2	6	-8

Negative excess demand means excess supply

b) Market-clearing price is the price at which demand is equal to the supply. Based on the calculated excess demand, the market-clearing price is ₹4 because at this price, the demand is equal to supply and hence market is cleared.

c) Given the market model:

$$D = 10 - 2P \text{-------(1)}$$
$$S = -6 + 2P \text{ ------(2)}$$
$$D = S \qquad \text{------(3)}$$

Substituting equation (1) and (2) in equation (3), we get

$$10 - 2P = -6 + 2P$$

$$=> \qquad 4P = 16$$

$$=> \qquad P = 4$$

Putting the value of P in equation (1) or (2) gives the quantity demanded and supplied. Therefore, $D = S = 2$

9.4 Given the demand and supply models

$$Demand: q = 100 - 5p$$

$$Supply: q = -10 + 4P$$

Find the equilibrium price and quantity if government imposes a sales tax of ₹4 per unit.

Solution:

Sales tax is a tax that is paid for sales of certain goods and services. This tax is to be paid by the suppliers. Therefore, the supply function will change.

This problem may be solved easily by using the inverse function.

Therefore,

The inverse demand function becomes

$$p = 20 - 0.2q - - - -(9.1)$$

And inverse supply function without tax becomes

$$p = 0.25q + 2.5 - - - -(9.2)$$

A sales tax of ₹4 per unit changes the inverse supply function to

$$p = 0.25q + 2.5 + 4$$

Or,

$$p = 0.25q + 6.5 - - - -(9.3)$$

In equilibrium, supply price is equal to demand price, therefore, equating equation (9.1) and (9.3)

$$20 - 0.2q = 0.25q + 6.5$$

$$0.45q = 13.5$$

$$q = \frac{13.5}{0.45} = 30 - - - -(9.4)$$

Substituting equation (9.4) in equation (9.1) gives the

$$p = 20 - 0.2(30)$$

Or,

$$p = 20 - 0.2(30) = 14$$

Similarly, substituting equation (9.4) in equation (9.3) gives

$$p = 0.25(30) + 6.5 = 14$$

Therefore, the equilibrium price = 14 and equilibrium quantity = 30

9.5. Given the following non-linear market model:

$$D = 30 - P^2 \text{ -------(1)}$$
$$S = P^2 - 2 \text{ ---------(2)}$$
$$D = S \text{ --------(3)}$$

Find the equilibrium price and quantity

Solution: Substituting equation (1) and (2) in equation (3), we get

$$30 - P^2 = P^2 - 2$$
$$=> \quad 2P^2 = 32$$
$$=> \quad P^2 = 16$$
$$=> \quad P = \sqrt{16}$$
$$=> \quad P = \pm 4$$

Since negative values do not carry economic meaning, $P = 4$

Substituting the value of P in equation (1) or (2) gives $D = S = 14$

9.6. Given the following non-linear market model:

$$D = 130 - 5P^2$$
$$S = -45 + 10P$$
$$D = S$$

Determine the equilibrium price and quantity.

Solution:

Given the non-linear market model

$$D = 130 - 5P^2 \text{---------(1)}$$
$$S = -45 + 10P \text{-------(2)}$$
$$D = S \text{ ---------(3)}$$

Substituting equation (1) and (2) in equation (3), we get

$$130 - 5P^2 = -45 + 10P$$

$$=> \; 5P^2 + 10P - 175 = 0 \text{------(4)}$$

Equation (4) is a quadratic equation. The solution for a quadratic equation of the form $ax^2 + bx + c = 0$ $(a \neq 0)$ is given by the following formula:

$$(x_1, x_2) = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Using this formula in equation (4), a= 5, b= 10 and c= -175

Therefore, the solution for equation (4) will be

$$(P_1,P_2) = \frac{-10 \pm \sqrt{10^2 - 4 \times 5 \times (-175)}}{2 \times 5}$$

$$= \frac{-10 \pm \sqrt{100+3500}}{10}$$

$$= \frac{-10 \pm \sqrt{3600}}{10}$$

$$= \frac{-10 \pm 60}{10}$$

$$\therefore P_1 = \frac{-10+60}{10} = 5$$

$$\text{and } P_2 = \frac{-10-60}{10} = -7$$

Since negative values do not carry economic meaning, equilibrium price = 5

Substituting the value of P in equation (1) or (2) we get equilibrium quantity = 5